Lecture 4 - January 19

Math Review

Propositional Logic, Predicate Logic

Announcement

- Tuesday's lecture recording mossing!
- Lab1 released
  + tutorial videos
  + problems to solve

Implication ≈ Whether a Contract is Honoured

$p \Rightarrow q$

p: promised term (e.g. salary)

q: obligations (e.g. work).

$p \Rightarrow q$ T if the contract is not breached

(C1) $p = T$ $q = T$ T

(C2) $p = T$ $q = F$ F.

(C3) $p = F$ $q = T$ T

(C4) $p = F$ $q = F$. T.

Common base/trivial cases for justification.

Expressing Implications

p: snow storm
q: cancel class

q if p, p is sufficient for q

casual relation

if p then q

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

when the cause is not T then it doesn't matter what consequence is (the causal rel. not violated)

q unless ¬p

conditions for ⇒ to be true

C1: q

C2: p = F

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

p only if q, q is necessary for p

q is necessary to be T, in order for ⇒ to be T if we know p is T

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

known: p T

the only way the ⇒ is T is q is also T

don't care what q is.

$$\boxed{p \Rightarrow q \equiv \neg p \vee q}$$

Which of the following expressions are equivalent to $p \Rightarrow q$

(1) q if p ✓

(2) q only if p X

$\Longleftrightarrow$

$p \Longleftrightarrow q$

$\neg(p \wedge q) = \neg p \vee \neg q$

$\neg(p \vee q) = \neg p \wedge \neg q$

$\neg(x < 0) \to x \geq 0$

$\therefore\ p \Rightarrow q$

Contrapositive (Converse of Inverse)

$y < 23 \wedge y \geq 46 \Rightarrow x \leq 0 \vee x > 23$

e.g. $x > 0 \wedge x \leq 23 \Rightarrow y \geq 23 \vee y < 46$

Converse

$y \geq 23 \vee y < 46 \Rightarrow x > 0 \wedge x \leq 23$

Inverse: $\neg p \Rightarrow \neg q$

equivalence proof in equational style

$\neg(x > 0 \wedge x \leq 23) \Rightarrow \neg(y \geq 23 \vee y < 46)$

$\equiv$ { De Morgan }

$x \leq 0 \vee x > 23 \Rightarrow y < 23 \wedge y \geq 46$

Prove $\quad p \Leftrightarrow q$

if $\quad$ only if

Need to proof

(1) $\quad p \Leftarrow q \qquad p$ if q

$q \Rightarrow p$

(2) $\quad p \Rightarrow q \qquad p$ only if q

Identity

identities $\boxed{\begin{array}{l} 0 + a = a \\ 1 * a = a \end{array}}$

$True \Rightarrow P \equiv P$

as if: $(True \Rightarrow P) \equiv P$

$True \wedge P \equiv P$

$false \vee P \equiv P$

Precedence of operators

$\neg$

$\wedge$

$\vee$

$\Rightarrow$

$\equiv$

Zero

$false \Rightarrow P \equiv True$

$false \wedge P \equiv \underline{false}$

$true \vee P \equiv true$

Predicate Logic: Quantifiers

- syntax
- base cases in programming

$$\forall i \bullet R(i) \Rightarrow P(i)$$

universal

1) i in the range, then $P(i)$ must be true

2) i not in the range, then ~~don't care~~ zero of false $\Rightarrow P \equiv$ True

empty array means $R(i)$ is false ↳ false $\Rightarrow P \equiv T$

$$\exists i \bullet R(i) \wedge P(i)$$

existential

Range

empty array means $R(i)$ is false (universe of disclosure) ↳ false $\wedge P \equiv F$

$\forall i \bullet P(i)$

$\exists i \bullet P(i)$

∵ no counter-example or witness of violation can be found.

universal property

```
boolean allPos(int[] a)
{
  if(a.length == 0){
    return true;
  }
}
```

existential property

```
boolean somePos(int[] a)
{
  if(a.length == 0){
    return false;
  }
}
```

∵ no witness of satisfaction can be found.

Rodin

$\forall$

$!x : R(x) \Rightarrow P(x)$

$\exists$ $\#x : R(x) \land P(x)$

✓

TLA+

$R(x)$ $\Rightarrow$

\A x \in Nat, y \in Int : P(x)

$\land$

\E x \in Nat, y \in Int : P(x)

$\mathbb{N}$ natural #'s

↳ $0, 1, 2, 3, \ldots +\infty$

$\mathbb{Z}$ integers

↳ $-\infty, \ldots, 0, \ldots +\infty$

Logical Quantifiers: Examples

$\forall i \bullet i \in \mathbb{N} \Rightarrow i \geq 0$

$\mathbb{N} = 0, 1, 2, \ldots$

all elements in range ≥ 0

$\forall i \bullet i \in \mathbb{Z} \Rightarrow i \geq 0$

witness/counter-example : -2

$-2 \in \mathbb{Z} \Rightarrow -2 \geq 0 \equiv T \Rightarrow F \equiv F$

$\forall i, j \bullet i \in \mathbb{Z} \wedge j \in \mathbb{Z} \Rightarrow i < j \vee i > j$

witness: $i = 2$, $j = 2$

$\underbrace{\underbrace{2 \in \mathbb{Z}}_{T} \wedge \underbrace{2 \in \mathbb{Z}}_{T}}_{T} \Rightarrow \underbrace{\underbrace{2 < 2}_{F} \vee \underbrace{2 > 2}_{F}}_{F}$

F

$\exists i \bullet \underbrace{i \in \mathbb{N}}_{0 \in \mathbb{N}} \wedge \underbrace{i \geq 0}_{}$ $\quad T \wedge T \equiv T$

witness: 0

$\exists i \bullet i \in \mathbb{Z} \wedge i \geq 0$

witness : 0

$\exists i, j \bullet i \in \mathbb{Z} \wedge j \in \mathbb{Z} \wedge (i < j \vee i > j)$

$i = 1$, $j = 3$